

Mathlete Training Centre

**APMOPS 2026 Paper**

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1. Two positive integers satisfy that five-sixths of one integer equals one-fourth of the other integer. Find the minimum possible value of the sum of these two integers.

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2. A factory's total income last year exceeded its total expense by \$12000. It is estimated that this year, the total income will increase by 30% compared to last year, while the total expense will decrease by 20%. As a result, this year's total income will exceed the total expense by \$20000. What was the total income of the factory last year?

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3. There are two circles whose difference in areas is 169 square centimeters. It is known that the ratio of their circumferences is 7 : 6. What is the area of the smaller circle in square centimeters?

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4. When 246 is added to its reverse 642, the sum is 888. Find the largest integer such that the sum of the integer and its reverse is 786. (Note: The leading digit of the integer and its reverse cannot be 0.)

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5. Towns  $A$ ,  $B$ ,  $C$ , and  $D$  lie in this order along a straight road. It is known that the distance from  $A$  to  $B$  is  $\frac{3}{5}$  of the distance from  $C$  to  $D$ , and the distance from  $A$  to  $C$  is  $\frac{6}{7}$  of the distance from  $B$  to  $D$ . If the distance from  $B$  to  $C$  is 36 km, find the distance from  $A$  to  $D$  (in km).

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6. Two candles with the same thickness and material have initial lengths in the ratio 19 : 15. They are lit at the same time and burn at the same constant rate. After 34 minutes of burning, the ratio of their remaining lengths becomes 10 : 7. How many more minutes does the longer candle need to burn completely?

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7. Five distinct integers are selected from the set  $\{1, 4, 7, 10, 13, 16, 19, 22, 25, 28\}$  such that the difference between any two selected numbers is not divisible by 5. How many different selections are possible?

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8. Let  $x, y, z$  be positive integers satisfying the equation  $39x + 40y + 41z = 567$ . Find the value of  $29x + 30y + 31z$ .

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9. The first two terms of a sequence are 2016 and 2026. Starting from the third term, each term is the average of the previous two terms:

2016, 2026, 2021, 2023.5, ...

Write down the integer closest to the 19<sup>th</sup> term of this sequence.

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10. Let  $A$  and  $B$  be positive integers with  $B \neq 1$ . If both  $A + B$  and  $A + B^2$  are perfect squares, find the smallest possible value of  $B$ .

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11. Evaluate:  $128 \times \left( \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{24} + \frac{1}{48} + \frac{1}{96} + \frac{1}{192} + \frac{1}{384} \right)$ .

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12. In a factory workshop, there are three workers. They are producing the same type of part. Worker A takes 6 minutes per part, Worker B takes 7 minutes per part, and Worker C takes 8 minutes per part.

They are assigned to jointly produce a total of 3577 parts. If all three workers start working at the same time, how many parts will Worker A have produced when the entire batch is completed?

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13. There are two construction projects,  $A$  and  $B$ , assigned to Team A and Team B respectively. Team A is responsible for project  $A$ , and Team B is responsible for project  $B$ .

On a sunny day, Team A can complete project  $A$  in 12 days, while Team B can complete project  $B$  in 15 days. However, during the construction period, there were several rainy days. On rainy days, Team A's efficiency decreases by 40% (i.e., becomes 60% of normal), while Team B's efficiency decreases by 10%.

Both teams start working at the same time and eventually complete their respective projects on the same day. How many rainy days were there during the construction period?

14. A positive integer  $N$  is the product of two prime numbers. It is given that the sum of all positive divisors of  $N$  is 3042. Find the value of  $N$ .

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15. Find the number of four-digit positive integers satisfying the following condition: each of the first three digits is divisible by the last digit.

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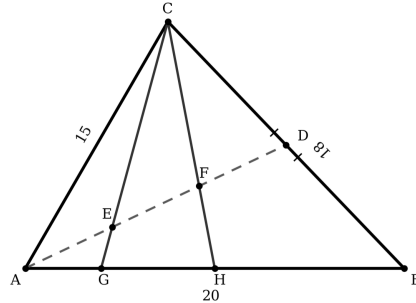
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16. It is known that  $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{1}{4}n^2(n+1)^2$ . Find the value of  $1^3 - 2^3 + 3^3 - 4^3 + \dots + 19^3 - 20^3 + 21^3$ .

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17. In the figure below, triangle  $ABC$  has side lengths  $AB = 20$ ,  $BC = 18$ , and  $CA = 15$ . Point  $D$  is the midpoint of  $BC$ . Points  $E$  and  $F$  trisect the line segment  $AD$ . The lines  $CE$  and  $CF$  intersect  $AB$  at points  $G$  and  $H$ , respectively. Find the length of segment  $GH$ .



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18. A, B, and C are playing table tennis. In each game, only two players compete. After each game, the loser leaves and is replaced by the third player who was not playing.

At a certain moment, it is known that A has played 9 games in total and B has played 6 games in total. What is the maximum possible number of games that C could have played?

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19. A bag contains 6 red candies and 3 yellow candies. Jamie randomly selects one candy from the bag each time and eats it immediately.

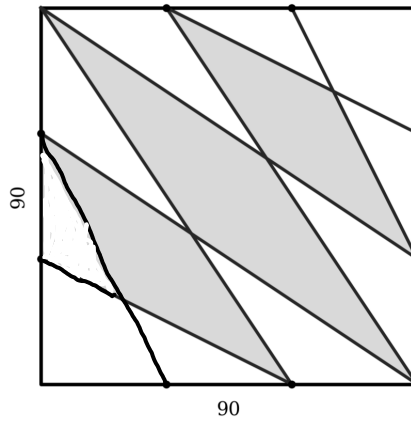
After she has eaten 4 candies in total, the probability that she never ate a yellow candy immediately after finishing a red candy is  $\frac{m}{n}$ , where  $m$  and  $n$  are coprime positive integers.

Find the value of  $m + n$ .

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20. The diagram shows a  $90 \times 90$  square, and each side is divided into three equal parts. Some of these division points are connected in the figure. Find the total area of the shaded region.



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21. A seven-digit number formed using each of the digits 1, 2, 3, 4, 5, 6, 7 exactly once is called a “valley number” if its digits strictly decrease from the beginning to the smallest digit and then strictly increase to the end.

For example, 4312567 is a valley number, while 1234567 is not.

How many different valley numbers can be formed?

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22. Find the largest prime number that divides

$$1 \times 2 \times 3 + 2 \times 3 \times 4 + \cdots + 2023 \times 2024 \times 2025 + 2024 \times 2025 \times 2026.$$

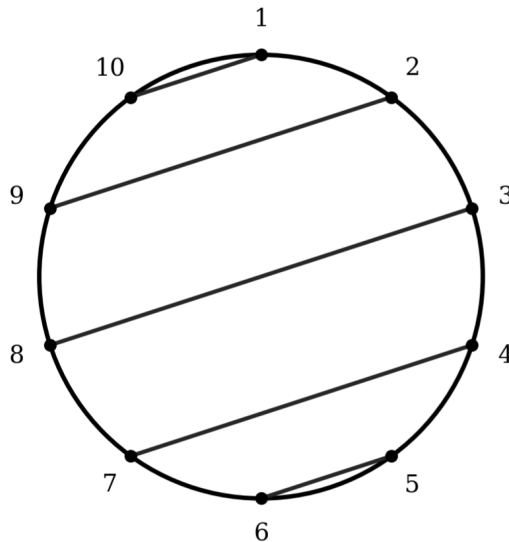
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23. There are 10 distinct points on a circle. We want to pair up these points and connect each pair with a line segment such that no two segments intersect.

For example, the figure on the right shows one such pairing method.

How many different non-intersecting pairing configurations are there?



24. A library has 100 books in total. System records show that A, B, and C borrowed 33, 44, and 55 books respectively. Among them, A and B both borrowed 29 books; A and C both borrowed 25 books; B and C both borrowed 36 books.

What is the maximum possible number of books that were never borrowed by any of the three people?

25. Let  $x, y, z, w$  be four positive integers satisfying the following conditions:

- The pairs  $(x, y)$ ,  $(y, z)$ , and  $(z, w)$  are not coprime;
- All other pairs among these four numbers are coprime.

Find the minimum value of  $x + y + z + w$ .

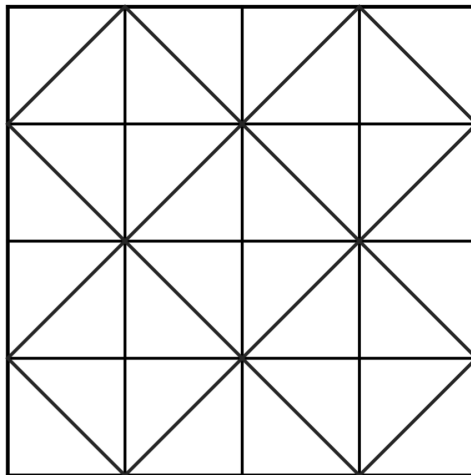
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26. How many triangles are there?



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27. How many six-digit numbers are there such that the product of their digits equals  $6^6$ ?

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28. An isosceles triangle  $ABC$  has side lengths  $AB = 6$ ,  $CA = 5$ , and  $CB = 5$ . A point  $P$  is chosen randomly inside the triangle.

The probability that  $PC < PB < PA$  is  $\frac{m}{n}$ , where  $m$  and  $n$  are coprime positive integers.

Find the value of  $m + n$ .

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29. For any positive integer  $m, n$ , define  $r(m, n)$  as the remainder of  $m \div n$  (for example,  $r(8, 3) = 2$ ), then the smallest positive integer solution of  $m$  that satisfies the equation

$$r(m, 1) + r(m, 2) + r(m, 3) + \cdots + r(m, 10) = 4$$

is \_\_\_\_\_.

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30. An arithmetic sequence is a sequence in which the difference between any two consecutive terms is constant. If an 8-term arithmetic sequence contains both 20 and 26, then it is called a “good sequence”.

For example,  $(18, 20, 22, 24, 26, 28, 30, 32)$  and  $\left(26, 24\frac{1}{2}, 23, 21\frac{1}{2}, 20, 18\frac{1}{2}, 17, 15\frac{1}{2}\right)$  are both good sequences.

Find the sum of all possible first terms  $a_1$  of all good sequences  $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ .

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