Junior Mathematical Olympiad

Tuesday 11 June 2024

England & Wales: Year 8 or below
Scotland: S2 or below
Northern Ireland: Year 9 or below

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of rulers, set squares and compasses is allowed, but **calculators and protractors are forbidden**. You are strongly encouraged to use geometrical instruments to construct large, accurate diagrams for geometry problems.
4. Start each question on an official answer sheet on which there is a **QR code**.
5. If you use additional sheets of (plain or lined) paper for a question, please write the following in the top left-hand corner of each sheet. (i) The question number. (ii) The page number for that question. (iii) The digits following the ':' from the question's answer sheet QR code.
   Please do not write your name or initials on additional sheets.
6. **Write on one side of the paper only**. Make sure your writing and diagrams are clear and not too faint. (Your work will be scanned for marking.)
7. **Arrange your answer sheets in question order before they are collected**. If you are not submitting work for a particular problem, please remove the associated answer sheet.
8. **Your answers should be fully simplified, and exact**. They may contain symbols such as π, fractions, or square roots, if appropriate, but not decimal approximations.
9. **Section A** - Only answers are required.
10. **Section B** - You should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
11. **Scoring** Each question in Section A is worth 1 point. Each question in Section B is worth up to 10 points. Answers in Section B can be awarded partial marks depending on on the clarity of the participant's mathematical presentation
12. To accommodate candidates sitting in other time zones, please do not discuss the paper on the internet until **8am GMT on Thursday 13 June**, when the solutions video will be released at ukmt.org.uk/competitions-papers. Candidates in time zones more than 5 hours ahead of GMT must sit the paper on Wednesday 12 June (as defined locally).

Section A

Try to complete Section A within 30 minutes or so. Only answers are required.

A1. What is the sum of the first nine primes?

A2. Forty-two cubes of side-length 1 cm are stuck together to form a solid cuboid. The perimeter of the base of the cuboid is 16 cm. What is its height, in cm?

A3. $PQRS$ is a square which has been divided into four regions: two identical rectangles, one square of area 9 cm^2 and a second square of area 25 cm^2. What is the area of square $PQRS$, in cm^2?

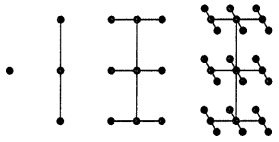
A4. Note that $49 = 4 \times 9 + 4 + 9$. How many two-digit numbers are equal to the product of their digits plus the sum of their digits?

A5. The difference between an interior angle of a regular polygon and an exterior angle of the same polygon is 150°. How many sides does the polygon have?

A6. When a group of five friends met up, Alice shook hands with one person; Bill shook hands with two people; Cara shook hands with three people; Dhriti shook hands with four people. How many people did Erin shake hands with?

A7. The time is 20:24 (expressed in 24-hour time). What is the angle between the hour hand and the minute hand on an accurate analogue clock, in degrees?

A8. We make a sequence of diagrams. The first diagram consists of a single node. The second diagram is made from the first diagram by drawing two edges of length 1 cm from that node, and putting a node at the other end of each new edge. After that, we make each new diagram from the previous diagram by adding two new edges to each node, with these new edges each having half the length of the edges that were added in the previous diagram. We also attach a node to the end of each new edge. The first four diagrams are shown. Find the total length of all the edges in the fifth diagram, in cm.



A9. The numbers x and y satisfy the equations:

$$xy = \frac{7}{6} \qquad x(y+1) = \frac{5}{3} \qquad y(x+1) = \frac{7}{2}$$

What is the value of $(x+1)(y+1)$?

A10. What is the last digit of $2^{(2^{2024})}$?

Section B

Your solutions to Section B will have a major effect on your result.

Concentrate firstly on one or two Section B questions and then write out *full solutions* (not just brief 'answers'), including mathematical reasons as to why your method is correct.

You will have done well if you hand in full solutions to two or more Section B questions.

Do *not* hand in rough work.

B1. What is the smallest positive integer that only contains the digits 0 and 1, and is divisible by 36?

B2. Natasha and Rosie are running at constant speeds in opposite directions around a running track. Natasha takes 70 seconds to complete each lap of the track and meets Rosie every 42 seconds.

How long does it take Rosie to complete each lap?

B3. The positive integers from 1 to n ($n \geq 2$) inclusive are to be spaced equally around the circumference of a circle so that:

(a) no two even numbers are adjacent;

(b) no two odd numbers are adjacent;

(c) no two numbers differing by 1 are adjacent.

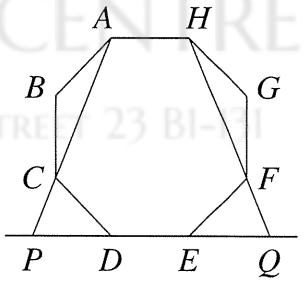
What is the smallest value of n for which the above is possible?

B4. My piggy bank contains x pound coins and y pennies and rattles nicely. If instead it contained y pound coins and x pennies, then I would only have half as much money.

What is the smallest amount of money my piggy bank could contain?

B5. A regular octagon $ABCDEFGH$ has sides of length 1. The lines AC and HF meet the line going through D and E at P and Q respectively.

What is the length of the line PQ?



B6. Let A be the set of $2n$ positive integers $1, 2, 3, \ldots, 2n$, where $n \geq 1$.

For which values of n can this be split into n pairs of integers in such a way that every pair has a sum which is a multiple of 3?

As always in Olympiad problems such as this, you also need to explain why no other values of n are possible.