Intermediate Mathematical Olympiad

MACLAURIN PAPER

Thursday 21 March 2024

England & Wales: Year 11 | *Scotland: S4* | *Northern Ireland: Year 12*

These problems are meant to be challenging.

Try to finish whole questions even if you cannot do many; you will have done well if you hand in a complete solution to two or more questions.

Instructions

1. Time allowed: **2 hours**.
2. **Full written solutions – not just answers – are required**, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.
3. **One complete solution will gain more credit than several unfinished attempts**.
4. **Each question carries 10 marks**.
5. The use of rulers, set squares and compasses is allowed, but **calculators and protractors are forbidden**.
6. Start each question on an official answer sheet on which there is a **QR code**.
7. If you use additional sheets of (plain or lined) paper for a question, please write the following in the top left-hand corner of each sheet. (i) The question number. (ii) The page number for that question. (iii) The digits following the ':' from the question's answer sheet QR code. **Please do not write your name or initials on additional sheets**.
8. **Write on one side of the paper only**. Make sure your writing and diagrams are clear and not too faint. (Your work will be scanned for marking.)
9. **Arrange your answer sheets in question order before they are collected**. If you are not submitting work for a particular problem, please remove the associated answer sheet.
10. To accommodate candidates sitting in other time zones, please do not discuss the paper on the internet until **8am GMT on Saturday 23 March**.
11. Do not turn over until told to do so.

Advice to candidates

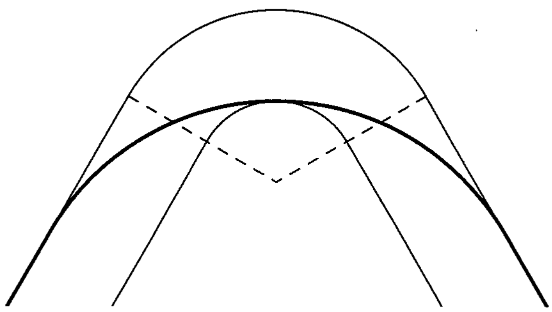
- ◇ *Do not hurry, but spend time working carefully on one question before attempting another.*
- ◇ *Try to finish whole questions even if you cannot do many.*
- ◇ *You will have done well if you hand in full solutions to two or more questions.*
- ◇ *Your answers should be fully simplified, and exact. They may contain symbols such as* π*, fractions, or square roots, if appropriate, but* not *decimal approximations.*
- ◇ *Give full written solutions, including mathematical reasons as to why your method is correct.*
- ◇ *Just stating an answer, even a correct one, will earn you very few marks.*
- ◇ *Incomplete or poorly presented solutions will not receive full marks.*
- ◇ *Do* not *hand in rough work.*

1. Mike is doing a one-hour cycling challenge. He has a computer which predicts how far he will cycle in the rest of the hour based on his average speed so far.
After cycling 1 km in t minutes, he checks the distance the computer predicts he will cycle in the remaining time and it shows d km.
In the next 36 minutes, he cycles 15 km. He checks the computer again and finds it still predicts he will cycle d km in the remaining time.
Find the distance shown on the computer each time he looks.

2. In how many ways can we choose two different integers between -100 and 100 inclusive, so that their sum is greater than their product?

3. What is the smallest number, n, which is the product of 3 distinct primes where the mean of all its factors is not an integer?

4. A bend in a road is formed from two concentric arcs with inside radius r and outside radius R, each of a third of a circle with the same centre. The road is then formed of tangents to the arcs.
A cyclist cuts the corner by following an arc of radius x which is tangent to the outside of the road at its ends and tangent to the inside of the road in the middle.



Prove that $r + x = kR$ for some number k to be found.

5. Two right-angled triangles are similar. The larger triangle has short sides which are 1 cm and 5 cm longer than the short sides of the smaller triangle. The area of the larger triangle is $8\,\text{cm}^2$ more than the area of the smaller triangle. Find all possible values for the side lengths of the short sides of the smaller triangle.

6. A busy bee buzzes between the cells of a large honeycomb made up of a plane of tessellated hexagons. A flight of length n consists of picking any of the six neighbouring cells and flying to the n^{th} cell in that direction. After consecutive flights of lengths $n = N, N-1, \ldots, 2, 1$, the bee finds that it has returned to its starting location. For which values of N is this possible?