Intermediate Mathematical Olympiad

Cayley paper

Thursday 21 March 2024

England & Wales: Year 9 or below	*Scotland: S2 or below*	*Northern Ireland: Year 10 or below*

These problems are meant to be challenging.

Try to finish whole questions even if you cannot do many; you will have done well if you hand in a complete solution to two or more questions.

Instructions

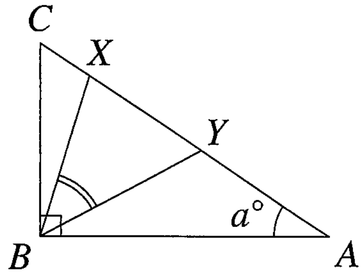
1. Time allowed: **2 hours**.
2. **Full written solutions – not just answers – are required**, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.
3. **One complete solution will gain more credit than several unfinished attempts**.
4. **Each question carries 10 marks**.
5. The use of rulers, set squares and compasses is allowed, but **calculators and protractors are forbidden**.
6. Start each question on an official answer sheet on which there is a **QR code**.
7. If you use additional sheets of (plain or lined) paper for a question, please write the following in the top left-hand corner of each sheet. (i) The question number. (ii) The page number for that question. (iii) The digits following the ':' from the question's answer sheet QR code. **Please do not write your name or initials on additional sheets**.
8. **Write on one side of the paper only**. Make sure your writing and diagrams are clear and not too faint. (Your work will be scanned for marking.)
9. **Arrange your answer sheets in question order before they are collected**. If you are not submitting work for a particular problem, please remove the associated answer sheet.
10. To accommodate candidates sitting in other time zones, please do not discuss the paper on the internet until **8am GMT on Saturday 23 March**.
11. Do not turn over until told to do so.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

Advice to candidates

- ◇ *Earlier questions tend to be easier. In general, you are advised to concentrate on these problems first. It is more important to complete a small number of questions than to try all the problems.*
- ◇ *Spend time working carefully on one question before attempting another.*
- ◇ *Numerical answers should be fully simplified and exact. They may contain symbols such as π, fractions or square roots, but not decimal approximations.*
- ◇ *Stating answers, even correct answers, without justification will earn you very few marks.*
- ◇ *You are encouraged to use geometrical instruments to construct large, accurate diagrams for geometry problems.*

1. A triangle, ABC, has a right angle at B and $\angle A = a°$. X lies on AC such that $AX = AB$. Y lies on AC such that $CY = CB$. Prove that $\angle XBY$ has the same size independently of the other angles of the triangle and find the size of that angle.



2. The Intermediate Maths Challenge has 25 questions with the following scoring rules:
5 marks are awarded for each correct answer to Questions 1-15;
6 marks are awarded for each correct answer to Questions 16-25;
Each incorrect answer to Questions 16-20 loses 1 mark;
Each incorrect answer to Questions 21-25 loses 2 marks.
Where no answer is given 0 marks are scored.
Fiona scored 80 marks in total. What possible answers are there to the number of questions Fiona answered correctly?

3. A positive integer $N < 2024$ is divisible by 39 times the sum of its digits. Find all possibilities for N.

4. When written in ascending order, the nine internal angles from three particular triangles form a sequence where the difference between any adjacent pair of numbers in the sequence is a constant d. One of the angles measures 42°. Find all possible values of the size of the largest of the nine angles.

5. A large number of people arrange themselves into groups of 2, 6 or 10 people. The mean size of a group is 5. However, when each person is asked how many other people are in their group (excluding themselves), the mean of their answers is 7. Prove that there are no groups of 6 people.

6. Into each row of a 9×9 grid, Nigel writes the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 in order, starting at one of the digits and returning to 1 after 9: for example, one row might contain 7, 8, 9, 1, 2, 3, 4, 5, 6. The grid is *gorgeous* if each nine-digit number read along a row or column or along the diagonal from the top-left corner to the bottom-right corner or the diagonal from the bottom-left corner to the top-right corner is divisible by 9. How many of the 9^9 possible grids are *gorgeous*?

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