Mathlete Training Centre
RMO

2024 Round 2 - Open

1. Calculate: $\dfrac{1\times 2+2\times 3+3\times 4+\ldots+44\times 45}{15}$

$$= \frac{1^2+1+2^2+2+3^2+3+\cdots+44^2+44}{15}$$

$$= \frac{1^2+\cdots+44^2+1+\cdots+44}{15} = \frac{\frac{44\times 45\times 89}{6} + \frac{44\times 45}{2}}{15}$$

$$= \frac{44}{2}\times(89+3) = 22\times 92 = 2024$$

2. Find the sum of digits for $\underbrace{333\ldots33}_{2024\text{ digits}}\times 2024$

$$= \frac{1}{3}\times 99\cdots9\times 2024$$

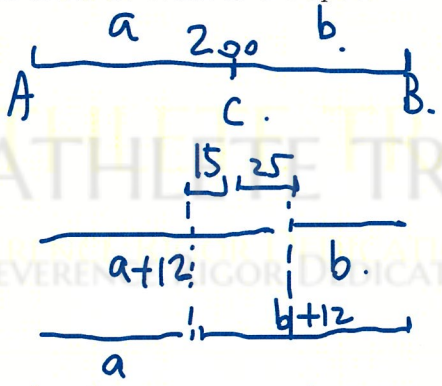
$$= \frac{1}{3}\times\left(2024\underbrace{00\cdots0}_{2024}-2024\right)$$

$$= \frac{1}{3}\times(202399\cdots97976)$$

$$= 6746\underbrace{\cdots6}_{2020}5992$$

$$\text{digit sum} = 6+7+4+6\times 2020+5+9+9+2$$
$$= 17+12120+25 = 12162$$

3. Andrew and Bill start from the Town A and Town B at the same time, travelling towards each other at constant speed and meet at Town C. The distance between Town A and Town B is 200km. If Andrew increased his speed by 12 km/h, they would meet at the ~~same~~ place 25km away from town C. If Bill increased his speed by 12km/h, they would meet at the place 15km away from town C. What is the speed of Bill?



$b = 25 \div \frac{5}{6} = 30 \text{ km/h}$ //

$12u = 15 + 25 = 40$] $\times 5$

$a + b + 12u = 200$

$a + b + 12u = 60u$

$a + b = 48 \text{ km/h}$

$200 \div 48 = 25 \div b = \frac{25}{6}$ hrs.

$200 \div 60 = \frac{10}{3}$ hrs.

$\frac{25}{6} - \frac{10}{3} = \frac{5}{6}$ hrs.

4. There are two 2-digit numbers. Charlie wanted to find their product but forgot the multiplication sign and obtained a 4-digit number N that is two times the correct product. Find the value of N.

$2 \times \overline{ab} \times \overline{cd} = \overline{abcd} = 100\,\overline{ab} + \overline{cd}$.

$(2 \times \overline{ab} - 1) \times \overline{cd} = 100\,\overline{ab}$. ①

$10 \leq \overline{ab} \underbrace{(2 \times \overline{cd} - 100)}_{\text{1 digit}} = \overline{cd} \leq 99$. ②

$C = 5$

$0 < d < 5$.

Ans 1352 //

$\overline{cd}$ is an even multiple of $\overline{ab}$.

$\overline{cd} = 2\overline{ab}$ or $\overline{cd} = 4\overline{ab}$ or $\overline{cd} = 6\overline{ab}$ or $\overline{cd} = 8\overline{ab}$

by ① $2\overline{ab} - 1$ odd multiple of 25 since $\frac{\overline{cd}}{\overline{ab}}$ is even.

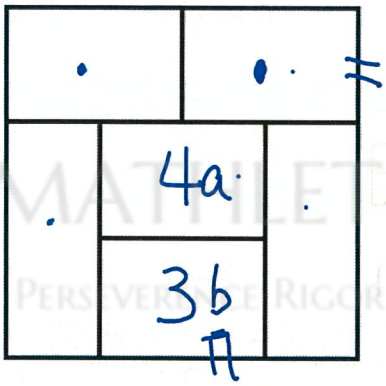
$= 25$ or 75 or ~~125~~ or ~~175~~

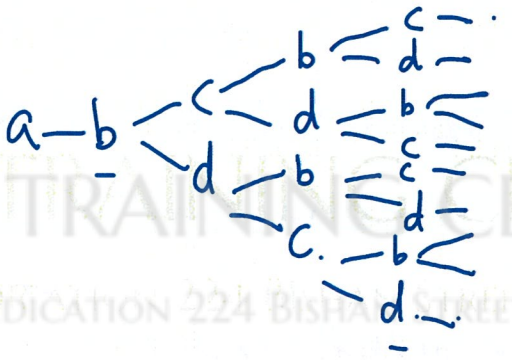
$\overline{ab} = 13$ or 38 sub into ① $25\,\overline{cd} = 100 \times 13$

$\overline{cd} = 52$.

$75\,\overline{cd} = 100 \times 38$ no soln

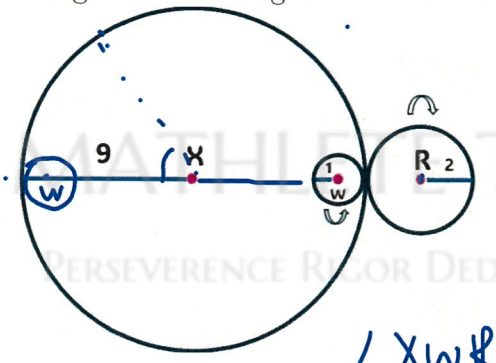
5. The figure below is a garden divided into 6 areas. There are 4 different tulips to be planted in the garden. Each area can only plant a tulip and two adjacent areas cannot have the same type of tulip. How many ways are there to plant the tulip?





$12 \times 10 = 120$

6. As shown in the figure below, Circle W, R and X each with radius of 1,2 and 9 respectively. Circle W rotate in anti-clockwise direction while Circle R rotate in clockwise direction. Find the angle XWR in degrees after circle W and R each turn 20 revolutions.



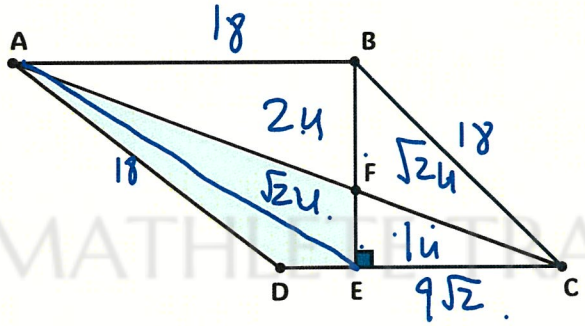
$20 \div 8 = 2 \text{ R } 4$

$2 \times 20 \div 11 = 3 \text{ R } 7$

$\angle XWR = \frac{7}{11} \times 360 - 180^\circ$

$= \frac{(7-5.5) \times 360}{11} = \frac{540}{11}$

7. As shown in the figure below, ABCD is a rhombus with AB= 18. △ BCE is an isosceles right triangle. Find the area of the shaded part



let $EC = a$

$a^2 + a^2 = 18^2 = 324$

$a^2 = 162 = 9^2 \times 2$.

$a = 9 \times \sqrt{2}$.

use butterfly model to cut ABCE into units of area, as labelled.

note. $\frac{EC}{AB} = \frac{1}{\sqrt{2}}$.

total area $= 18 \times 9\sqrt{2}$.

$= 2 \times (2+\sqrt{2})u = 2\sqrt{2}(1+\sqrt{2})u$

shaded $= (2+\sqrt{2}-1)u$

$= (1+\sqrt{2})u$.

$= 18 \times 9\sqrt{2} \div 2\sqrt{2}$

$= 81$ //

8. What is the maximum value of $10 - (x-1)(x-3)(x+3)(x+1)$ where x is a real number. (need solution)

$10 - (x^2-1)(x^2-9)$.

negative.

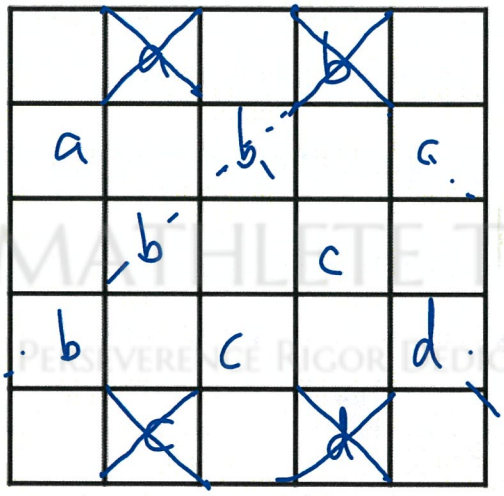
diff = 8.

when

$4 \times (-4) = -16$. $x^2 = 5$

$\max = 10 - (-16) = 10 + 16 = 26$ //

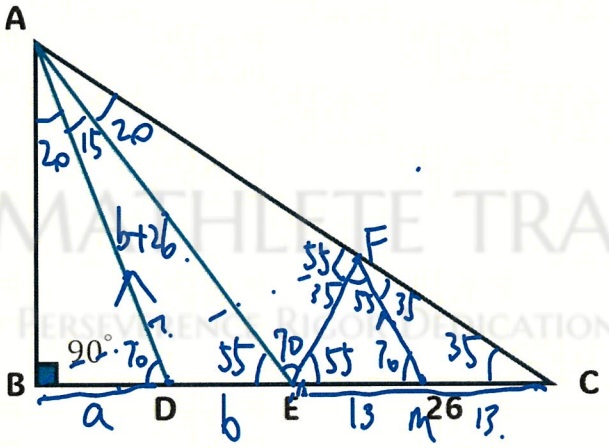
9. Shade the tiles in the grid below such that there are odd number of tiles shaded in each row, each column and each of the 18 diagonals. At most how many tiles can be shaded?



at most 21 tiles shaded

(need solution)

10. As shown in the figure below,△ ABC is a right angle triangle with $\angle B = 90°$ and $\angle BAC = 55°$. Points D and E are on BC such that $\angle BAD = 20°$ and $\angle BAE = 35°$. Given that EC=26, find the length of BD (need solution)



let M be midpt of EC.
F be foot of perpendicular from E to AC.
Then ME = MC = MF = 13
Join BF,
∠EFM = 55,
∠BFE = 35
⇒ ∠BFM = 90°.
also. ABF isosceles, AB = BF.
∠BAD = 20 = ∠FBM.
hence △ABD ≅ △BFM,
BD = MF = 13

11. A number is called a "Dragon Number" if the sum of its digits is a prime number. For example, 16 is a "Dragon Number" as 1+6=7,but 26 is not as 2+6=8. At most how many "Dragon Numbers" can be found in 5 consecutive integers? (need solution)

n
2 3 5 7
example of 4 dragon no.'s
199 200 201 202 203
19 2 3 4 5
289 290 291 292 293
19 11 12 13 14.

at most 1 pair of consecutive have carry over.
break into 2 groups from where the carry over happens.
One group has at least 3 numbers.
but there is no 3 consecutive prime numbers
⇒ at least one sum not prime.
↳ at most 4 //