Mathlete Training Centre
Round 2 RIPMWC Open

2014 RIPMWC Open Round 2 Answers

1) By successive division, $54+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{5}}}$

$x+y+z=2+3+5=10$

2) Let A's work rate be a per minute and B's work rate be b per minute.
$15a+9b=9a+27b$
$6a=18b \Rightarrow a=3b$
Hence B needs $(15-6)\times 3+9$ or $(9-6)\times 3+27=36$ minutes

3) $(3999\times 2014.2014+3999.3999\times 2014)\div 12.0012$
$=(3999\times 2014\times 1.0001+3999\times 2014\times 1.0001)\div(1.0001\times 12)$
$=[1.0001\times 2014\times(3999+3999)]\div(1.0001\times 12)$
$=1007\times 3999\times \frac{1}{3}$
$=1007\times(4000-1)\times \frac{1}{3}$
$=[4028000-1007]\times \frac{1}{3}$
$=1342331$

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4) The last digit has 4 choices: 0, 2, 4, 8
Case 1: The last digit is 0
The 1st 2 digits have $6\times 5=30$ even numbers
Case 2: The last digit is 2, 4 or 8
The last digit has 3 choices
The 1st and 2nd digits have 5 choices each
The number of even numbers $=3\times 5\times 5=75$
Hence the total numbe of 3-digit even numbers = 75 + 30 = 105

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5) Since $2014=287\times 7+5$. If Jack picks 4 coins, then the total number of coins left $=2014=287\times 7+1$. Then no matter how many coins Jane picks subsequently at each of her turn, Jack can match by picking $7-x$. So the game will end with Jane having to pick up the last coin.

6) The numbers are $2014, 2014+3, 2014+2(3), ..., 2014+16(3)$
The possible sum of 3 numbers range from $(2014+3)3 to (2014+45)3$
Number of different numbers $= 45 - 3 + 1 = 43$

7)

Pour no	Amount of milk in container A after the pour (cm^3)
1st	4030
2nd	$4030(1+\frac{1}{3} = 4030 \times \frac{4}{3})$
3rd	$4030 \times \frac{4}{3} \times \frac{3}{4} = 4030$
4th	$4030(1+\frac{1}{5}) = 4030 \times \frac{6}{5}$
5th	$4030 \times \frac{6}{5} \times \frac{5}{6} = 4030$

After the 2013th pour,
amount of milk in container A $= 4030 \times \frac{2014}{2013} \times \frac{2013}{2014} = 4030$
After the 2014th pour,
amount of milk in container A $= 4030 \times \left(1 + \frac{1}{2015}\right) = 4032$

8) Perimeter of $ABCEFGDA$ is $26\frac{2}{7}$.
Let x cm be the length of a side of the square.
$5x + \frac{\pi x}{2} = \frac{184}{7}$
$\frac{46}{7}x = \frac{184}{7} \Rightarrow x = 4$

Area of the shaded region $= \frac{1}{2}\left[4^2 + 4^2 + \frac{1}{2} \times \frac{22}{7} \times 2^2 - \frac{1}{2} \times 4 \times 10\right] = 9\frac{1}{7}\ cm^2$

9) $264 = 8 \times 3 \times 11$
N is divisible by 8, since 448 is divisible by 8.
To be divisible by 11 and 3 respectively,
n must be divisible by 11 and $26 + 7n$ must be divisible by 3.
A quick check yields $n = 286$

10) Consider AB together as 1 unit, there are 2 ways.
Number of ways of arranging 4 student and the unit of $AB = {}^5P_5 = 120$ ways
Total number of ways of arranging the 6 students $= 2 \times 120 = 240$ ways

11) $36 = 2^2 \times 3^2$
$42 = 2 \times 3 \times 7$
Hence LCM $= 2^3 \times 3^2 \times 7 = 252$
Number of pencil marks after 252 cm $= \dfrac{252}{36} + \dfrac{252}{42} - 1 = 7 + 6 - 1 = 12$
Circumference $= \dfrac{48}{12} \times 252 = 1008$ cm

12) 1, 2, 2, 2, 2, 2 Number of ways = ${}^6C_1$=6
1, 1, 1, 2 , 2, 2 Number of ways = ${}^7C_3 = 35$
and so on...

Hence total number of ways = ${}^6C_1 +^7 C_3 +^8 C_5 +^9 C_7 +^{10} C_9 +^{11} C_{11} = 6 + 35 + 56 + 36 + 10 + 1 = 144$

13) Area of $\triangle ABJ : BCIJ : CDHI : DEGH = 1 : 3 : 5 : 7$
Area of $\triangle AEG = \dfrac{16}{3} \times 60 = 320$ cm^2
Area of $\triangle AFG = \dfrac{1}{4} \times 320 = 80$ cm^2
Area of trapezium FGHP $= \dfrac{7}{16} \times 80 = 35$ cm^2

14) $$\frac{1}{53} + \frac{1}{53+106} + \frac{1}{53+106+159} + \ldots + \frac{1}{53+106+159+\ldots+1961+2014}$$
$$= \frac{1}{53}[\frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} + .. + \frac{1}{1+2+3+\ldots+37+38}]$$
$$= \frac{1}{53}\left[\frac{1}{\frac{1\times 2}{2}} + \frac{1}{\frac{2\times 3}{2}} + \frac{1}{\frac{3\times 4}{2}} + \frac{1}{\frac{38\times 39}{2}}\right]$$
$$= \frac{2}{53}\left[1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \ldots + \frac{1}{38} - \frac{1}{39}] = \frac{2}{53} \times \frac{38}{39}\right]$$
$$= \frac{76}{2067}$$

15) Writing down the last 2 digits of the powers of 2^n, one discovers that they are periodic with period of 20, if we ignore 2^1:
02, 02, 08, 16, 32, 64, 28, 56, 12, 24, 48, 96, 92, 84, 68, 36, 72, 44, 88, 76, 52, 04
Since $3^{2014} - 1 = (3^4)^{503}3^2 - 1 \equiv 8 \pmod{20}$, the last 2 digits are 12.

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