

Mathlete Training Centre
AMC 2022 12A

1. What is the value of

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$$

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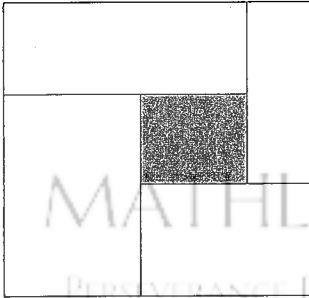
2. The sum of three numbers is 96. The first number is 6 times the third number, and the third number is 40 less than the second number. What is the absolute value of the difference between the first and second numbers?

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3. Five rectangles, A , B , C , D , and E , are arranged in a square as shown below. These rectangles have dimensions 1×6 , 2×4 , 5×6 , 2×7 , and 2×3 , respectively. (The figure is not drawn to scale.)

Which of the five rectangles is the shaded one in the middle?



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4. The least common multiple of a positive integer n and 18 is 180, and the greatest common divisor of n and 45 is 15. What is the sum of the digits of n ?

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5. The *taxicab distance* between points (x_1, y_1) and (x_2, y_2) in the coordinate plane is given by $|x_1 - x_2| + |y_1 - y_2|$. For how many points P with integer coordinates is the taxicab distance between P and the origin less than or equal to 20?

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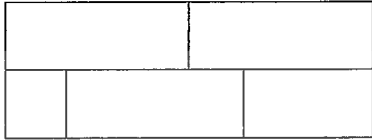
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6. A data set consists of 6 (not distinct) positive integers: 1, 7, 5, 2, 5, and X . The average (arithmetic mean) of the 6 numbers equals a value in the data set. What is the sum of all possible values of X ?

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7. A rectangle is partitioned into 5 regions as shown. Each region is to be painted a solid color — red, orange, yellow, blue, or green — so that regions that touch are painted different colors, and colors can be used more than once. How many different colorings are possible?



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8. The infinite product

$$\sqrt[3]{10} \cdot \sqrt[3]{\sqrt[3]{10}} \cdot \sqrt[3]{\sqrt[3]{\sqrt[3]{10}}}\dots$$

evaluates to a real number. What is that number?

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9. On Halloween 31 children walked into the principal's office asking for candy. They can be classified into three types: Some always lie; some always tell the truth; and some alternately lie and tell the truth. The alternaters arbitrarily choose their first response, either a lie or the truth, but each subsequent statement has the opposite truth value from its predecessor. The principal asked everyone the same three questions in this order.
- "Are you a truth-teller?" The principal gave a piece of candy to each of the 22 children who answered yes.
- "Are you an alternater?" The principal gave a piece of candy to each of the 15 children who answered yes.
- "Are you a liar?" The principal gave a piece of candy to each of the 9 children who answered yes.
- How many pieces of candy in all did the principal give to the children who always tell the truth?

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10. How many ways are there to split the integers 1 through 14 into 7 pairs so that in each pair the greater number is at least 2 times the lesser number?

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11. What is the product of all real numbers x such that the distance on the number line between $\log_6 x$ and $\log_6 9$ is twice the distance on the number line between $\log_6 10$ and 1?

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12. Let M be the midpoint of \overline{AB} in regular tetrahedron $ABCD$. What is $\cos(\angle CMD)$?

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13. Let R be the region in the complex plane consisting of all complex numbers z that can be written as the sum of complex numbers z_1 and z_2 , where z_1 lies on the segment with endpoints 3 and $4i$, and z_2 has magnitude at most 1. What integer is closest to the area of R ?

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14. What is the value of $(\log 5)^3 + (\log 20)^3 + (\log 8)(\log 0.25)$, where all logarithms have base 10?

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15. The roots of the polynomial $10x^3 - 39x^2 + 29x - 6$ are the height, length, and width of a rectangular box (right rectangular prism). A new rectangular box is formed by lengthening each edge of the original box by 2 units. What is the volume of the new box?

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16. A *triangular number* is a positive integer that can be expressed in the form $t_n = 1 + 2 + 3 + \dots + n$, for some positive integer n . The three smallest triangular numbers that are also perfect squares are $t_1 = 1 = 1^2$, $t_8 = 36 = 6^2$, and $t_{49} = 1225 = 35^2$. What is the sum of the digits of the fourth smallest triangular number that is also a perfect square?

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17. Suppose a is a real number such that the equation

$$a \cdot (\sin x + \sin(2x)) = \sin(3x)$$

has more than one solution in the interval $(0, \pi)$. The set of all such a can be written in the form $(p, q) \cup (q, r)$, where p, q, r are real numbers with $p < q < r$. What is $p + q + r$?

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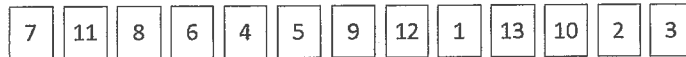
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18. Let T_k be the transformation of the coordinate plane that first rotates the plane k degrees counterclockwise around the origin and then reflects the plane across the y -axis. What is the least positive integer n such that performing the sequence of transformations $T_1, T_2, T_3, \dots, T_n$ returns the point $(1, 0)$ back to itself?

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19. Suppose that 13 cards numbered 1, 2, 3, ..., 13 are arranged in a row. The task is to pick them up in numerically increasing order, working repeatedly from left to right. In the example below, cards 1, 2, 3 are picked up on the first pass, 4 and 5 on the second pass, 6 on the third pass, 7, 8, 9, 10 on the fourth pass, and 11, 12, 13 on the fifth pass.



For how many of the $13!$ possible orderings of the cards will the 13 cards be picked up in exactly two passes?

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20. Isosceles trapezoid $ABCD$ has parallel sides \overline{AD} and \overline{BC} , with $BC < AD$ and $AB = CD$. There is a point P in the plane such that $PA = 1$, $PB = 2$, $PC = 3$, and $PD = 4$. What is $\frac{BC}{AD}$?

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21. Let $P(x) = x^{2022} + x^{1011} + 1$. Which of the following polynomials is a factor of $P(x)$?
- (A) $x^2 - x + 1$
 - (B) $x^2 + x + 1$
 - (C) $x^4 + 1$
 - (D) $x^6 - x^3 + 1$
 - (E) $x^6 + x^3 + 1$

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22. Let c be a real number, and let z_1 and z_2 be the two complex numbers satisfying the equation $z^2 - cz + 10 = 0$. Points z_1 , z_2 , $\frac{1}{z_1}$, and $\frac{1}{z_2}$ are the vertices of (convex) quadrilateral Q in the complex plane. When the area of Q obtains its maximum possible value, c is closest to which of the following?
- (A) 4.5
 - (B) 5
 - (C) 5.5
 - (D) 6
 - (E) 6.5

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23. Let h_n and k_n be the unique relatively prime positive integers such that

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} = \frac{h_n}{k_n}.$$

Let L_n denote the least common multiple of the numbers $1, 2, 3, \dots, n$. For how many integers n with $1 \leq n \leq 22$ is $k_n < L_n$?

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24. How many strings of length 5 formed from the digits 0, 1, 2, 3, 4 are there such that for each $j \in \{1, 2, 3, 4\}$, at least j of the digits are less than j ? (For example, 02214 satisfies this condition because it contains at least 1 digit less than 1, at least 2 digits less than 2, at least 3 digits less than 3, and at least 4 digits less than 4. The string 23404 does not satisfy the condition because it does not contain at least 2 digits less than 2.)

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25. A circle with integer radius r is centered at (r,r) . Distinct line segments of length c_i connect points $(0, a_i)$ to $(b_i,0)$ for $1 \leq i \leq 14$ and are tangent to the circle, where $a_i, b_i,$ and c_i are all positive integers and $c_1 \leq c_2 \leq \dots \leq c_{14}$. What is the ratio of $\frac{c_{14}}{c_1}$ for the least possible value of r ?

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