Mathlete Training Centre
Round 1 RIPMWC open

2010 RIPMWC open round 1

1.

Using only brackets, $+$, $-$, $\times$, $\div$, we can form the number 400 from multiple 4's as follows:

$$(4 \times 4 \times 4 + 4 \times (4 + 4) \div 4) \times 4$$

or

$$(4 \times 4 + 4) \times (4 \times 4 + 4)$$

Again using only brackets, $+$, $-$, $\times$, $\div$, What is the minimum number of 10's required to form the number 2010?

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2.

Each time the two hands of a standard 12-hour clock form a straight line, a bell chimes once. From 01 00 on a certain day to 01 00 the next day, how many chimes will be heard?

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3.

How many pairs of positive integers (m, n) satisfy the equation mn = 2010?

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4.

The sum of digits of 2010 is 2 + 0 + 1 + 0 = 3. The next number after 2010 that has the same sum of digits as 3 is 2100. Let a, b, c be the next 3 numbers after 2100 that has the same sum of digits as 3. What is the value of a + b + c?

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5.

Find the sum $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \frac{31}{32} + \frac{63}{64} + \frac{127}{128} + \frac{255}{256}$.

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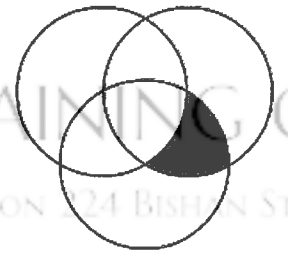
6.

As shown in the diagram below, each of the three circles with radius 2 cm passes through the centres of the other two circles. Taking π to be $\frac{22}{7}$, the area of the shaded region in cm^2 is



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7.

Suppose you are writing odd positive integers in a row, without blank spaces, as shown below

13579111315...

What will be the 1000th digit?

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8.

Four pupils Aaron, Bob, Charles and Dean took a mathematics examination in which each of their answer is either correct or wrong. Aaron got correct half of the total questions plus 7 questions, Bob got correct one third of the total questions plus 17 questions, Charles got correct one fourth of the total questions plus 22 questions and Dean got correct one fifth of the total questions plus 25 questions. There are between 1 and 100 questions in the examination. Which pupil got the most questions correct?

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9.

The last digit of $2008^3 - 2007^3 + 2006^3 - 2005^3 + \ldots + 2^3 - 1^3$ is

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10.

In the figure shown below, find $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G$.

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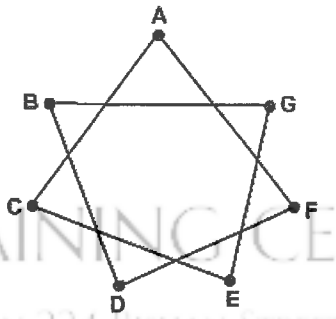

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11.

Find the value of the expression

$$\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{8}\right)\times\left(\frac{1}{2}+\frac{1}{3}+...+\frac{1}{9}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{9}\right)\times\left(\frac{1}{2}+\frac{1}{3}+...+\frac{1}{8}\right)$$

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12.

The sum of digits of the product $\underbrace{44...44}_{\text{Ten 4s}} \times \underbrace{199...998}_{\text{Nine 9s}}$ is

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13. There are 12 points on a circle such that the distances between adjacent pairs of points are all equal. How many right angled triangles can be formed by joining any 3 of the points?

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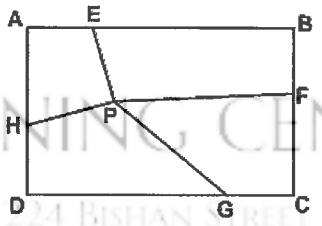
14. ABCD is a rectangle with AB = 8 cm and AD = 5 cm. Points E, F, G and H on AB, BC, DC and AD are such that AE = BF = GC = HD = 2 cm. P is a point in the interior such that the area of AEPH is 6 cm^2. What is the area of PFCG in cm^2 ?



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15. How many different words can you form from the letters ABCD, where a word is a sequence of one to four letters, using each letter at most once (for example DC and CADB)?

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16. A seven-digit telephone number $\overline{abcdefg}$ is called memorable if the number $\overline{abc}$ is the same as at least one of $\overline{def}$ or $\overline{efg}$. Assuming that every digit in the telephone number can range from 0 to 9, how many memorable telephone numbers are there?

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17. There are 8 consecutive traffic lights along a straight road. Each light remains green for 1.5 minutes, yellow for 4 seconds and red for 1.5 minutes. The lights are synchronised so that each light turns red 10 seconds after the preceding one turns red. What is the longest interval of time, in seconds, during which all the 8 lights are green?

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18. To decorate a rectangular notice-board which is $71\frac{1}{2}$ cm long and $40\frac{1}{3}$ cm wide, John is only allowed to use square papers. If it must be done such that the square papers do not overlap or extend beyond the edges of the notice-board, what is the minimum number of square papers needed?

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19.

A rectangle with the same perimeter as a square has $\frac{3}{4}$ its area. Find the ratio of the breadth of the rectangle to its length.

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20.

A car travels from town X to town Y along a highway. An hour after starting, it meets with an accident and is detained by the traffic police for half an hour, after which it proceeds at $\frac{3}{4}$ of its former speed and arrives $3\frac{1}{2}$ hours late. Had the accident happened 90 km farther along the highway, it would have arrived 3 hours late. Find the distance between town X and town Y in km.

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